Subject: Leaving Certificate Maths

Teacher: Mr Murphy

Lesson 3: Algebra

3.1 Learning Intentions

After this week's lesson you will be able to;

- Factorise a term in a number of methods
- Describe algebraic terms using the correct terminology Expand a binomial of power n

3.2 Specification

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
4.1 Expressions	– evaluate expressions given the value of the variables – expand and re-group expressions – factorise expressions of order 2 – add and subtract expressions of the form • $(ax+by+c) \pm \ldots \pm (dx+ey+f)$ • $(ax^2+bx+c) \pm \ldots \pm (dx^2+ex+f)$ where $a,b,c,d,e,f \in \mathbf{Z}$ • $\frac{a}{bx+c} \pm \frac{p}{qx+r}$ where $a,b,c,p,q,r \in \mathbf{Z}$ – use the associative and distributive properties to simplify expressions of the form • $a(bx \pm cy \pm d) \pm \ldots \pm e(fx \pm gy \pm h)$ where $a, b, c, d, e, f, g, h \in \mathbf{Z}$ • $(x \pm y)(w \pm z)$ – rearrange formulae	– perform the arithmetic operations of addition, subtraction, multiplication and division on polynomials and rational algebraic expressions paying attention to the use of brackets and surds – apply the binomial theorem

3.3 Chief Examiner's Report

Student should practise different ways of solving problesm; building up their arsenal of techniques on familiar problems will help them to tackle unfamiliar ones. Students at Higher level and Ordinary level should pay particular attention to algebraic methods of solving problesm as such methods are directly examinable at these levels.

3.4 Language in Algebra

Below are the key terms used in the topic of algebra. Variable:
This is usually shown as a letter that represents a number.
Coefficient: This is a number or symbol being multiplied by a variable
Constant: This is a number or value that does not change.
Expression: This is a combination of the above.

$$3x + 5$$

Now pause the video and highlight the key terms in the below expressions

$$\frac{2}{3}x - 4$$

$$\frac{x}{5}$$

Exponent: Is another term for power or index for example. $5x^2$

2 is the exponent here.

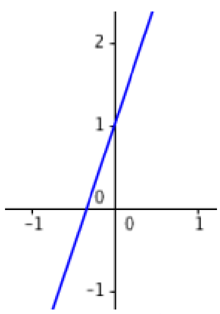
When we have a power in an expression, and this power is a whole number, we call this expression a polynomial.

$$3x^2 + 4x - 1$$

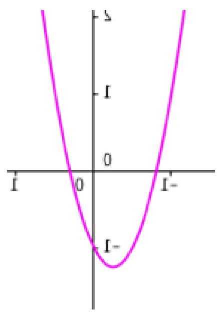
The highest power in the polynomial is referred to as the **degree**.

The degree is primarily responsible for 2 main ideas:

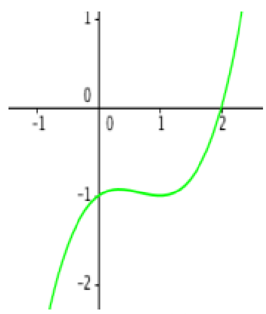
1. The shape of the polynomial if it is graphed as a function
2. The number of roots for that polynomial (more on this in strand 5)



Degree 1 = Linear



Degree 2 = Quadratic



Degree 3 = Cubic

3.5 Addition & Subtraction

We will look at both addition and subtraction together as they are what are described as Inverse Operations. This means that if we were to add a term to an expression, it can be undone by subtracting the same term. This idea is quite important when it comes to solving equations.

The most important aspect of addition and subtraction of algebra is the golden rule — ONLY terms with the same variables can be added/subtracted.

3.6 Multiplication

Unlike addition/subtraction, with multiplication we can multiply any term by any other term. We can show multiplication in a variety of ways:

5x. 3x 5x(3z) (5x)(3z) 5z x 3x

In order to multiply we;

1. Multiply the coefficients
2. Multiple like variables by adding their exponents
3. Clean up clean up

Here are some examples:

Term by term

Term by expression

$$5x(3x)$$
$$\Longrightarrow 15x^2$$

$$5x(3x^2 + 4y)$$

$$5x(3x^2) + 5x(4y)$$

$$15x^{1+2} + 20xy$$

$$15x^3 + 20xy$$

3.7 Division

Division is similar to multiplication in that we can divide any two terms.

The most conventional way to show division is:

$$\frac{6x^2}{2x} \ or \ {6x^2}/{2x}$$

However, we can express this using multiplication:

$$(6x^2)\frac{1}{2x} \ or \ (6x^2)(2x^{-1})$$

In order to divide we:

1. Divide the coefficients
2. Divide like variables by subtracting their powers
3. Gather all terms together

$$\frac{6x^2}{2x} = 3x^{2-1} = 3x^1$$

3.8 Expanding Brackets

This is where we have an expression multiplied by another expression or an expression raised to a power like:

$$(3x + 4)^2 = (3x + 4)(3x + 4)$$

There are two ways to do this, we can just multiply the two expressions as in section 2.6 or we can use a `cheat' to square an expression.

$$(3x + 4)^2 =$$

1. Square the first
2. Square the last
3. By Each Other and Double

However, this approach falters with

(3x + 4)6 = (3x + 4)(3x + 4) (3x + 4) (3x + 4) (3x + 4) (3x + 4)

To help with this we have Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Pascal's Triangle gives the coefficients of the terms in the expansion.

$$(3x + 4)^6$$

Our coefficients are:

1 6 15 20 15 6 1

Now we look after the terms in the expansion

For the first term, 3s, it begins with the power of the bracket, i.e. 6 And we reduce it by one as we move through the coefficients

1(3x)^6 6(3x)^5 15(3)^4 20(3x)^3 15(3z)^2 6(3x)^1 1(3x)^0

For the second term, 4, it begins with a power of 0,

and we increase it by one as we move through the coefficients

$$1(3x)^6(4)^0 + 6(3x)^5(4)^1 + 15(3x)^4(4)^2 + 20(3x)^3(4)^3 + 15(3x)^2(4^4) + 6(3x)^1(4)^5 + 1(3x)^0(4)^6$$

$$729x^6 + 5832x^5 + 19440x^3 + 34560x^3 + 34560x^2 + 18432x + 4096$$

3.9 Factorising

Factorising is the process of changing a term/expression into the factors that multiply together to give the original term/expansion. We have 3 main methods of factorising:

1. Highest Common Factor
2. Difference of 2 Squares
3. Quadratic Trinomials

Factorising-HCF

The highest common factor involves identifying the factors of the Terms in question and choosing the highest of these factors.

$$25x^2 + 10x$$
$$H.C.F. = 5x$$

$$5x(5x + 2)$$

Factorising-Difference of 2 Squares

This approach only works if we have:

1. Only two terms
2. Both terms are square terms
3. These terms are being subtracted

$$64x^2 - 49y^2$$
$$(8x)^2 - (7y)^2$$
$$(8x - 7y)(8x + 7y)$$

Factorising-Quadratic Trinomials

To identify a quadratic trinomial there should be:

1. Degree 2
2. Three terms*

*bear in mind that one term may have a coefficient of 0 and thus be invisible

$$x^2 + 4x + 3$$

The above is an example of a trinomial.

Guide Number Method

$$x^2 + 4x + 3$$
$$(\qquad)(\qquad)$$

3.10 Recap of Learning Intentions

After this week's lesson you will be able to;

- Factorise a term in a number of methods
- Describe algebraic terms using the correct terminology
- Expand a binomial of power n

3.11 Homework Task

Look at the following expressions and factorise/expand.

1) $6b(a^2b^2 - 3abc)$

2) $(3x^2 - 4xy)(2xy^2 - 4y)$

3) $\frac{12x^2yz^2}{2xyz} =$

Factorise:

5) $6xy + 10x^2y$

6) $49 - y^2$

7) $x^2 + 20x + 100$

8) $m^2 + 16m + 64$

9) $12t^2 + 11x + 2$

3.12 Solution to 2.11

Label the following functions as Injective, Surjective, Bijective or none of them. Be sure to use your graphing software to help with visualizing the functions. Be sure to write down the reason you have chosen your label. Be careful, one or two of these are tricky.

f (x) = x + 3 Bijective as each input has only one output (Injective) and every output has an input (Surjective)

g(x) = x^2 + 5x + 6 None, not every output has an input e.g. output of -100 (not surjective) 2 inputs can give the same output e.g. 2 and -2 (not injective).

$h(x) = \frac{1}{x+1}$ Injective, once we say the domain does not include -1. Not surjective as there is a missing x+i output (limit) where x=-1.

x^2 + y^2 = 25 not a function, this is a relation as one 'input' can give different 'outputs' (not really inputs/outputs, just used those terms to help with explanation) .

$j(x) = \frac{x^2+5x+6}{x+2}$ This is interesting. Do not simplify this. This function does not have a value at x = -2 as we x would end up dividing by zero. Therefore, similar reasoning to g(x), it is not surjective and is injective.